

1 ParameterLists Description

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Figure 1: Schematic of ParameterList heirarchy.

1.1 Integrator Base

Description:

Parent(s): ROOT

Child(ren): Integrator Settings (Section 1.2)
Integration Control Strategy Selection (Section 1.6)
Stepper Settings (Section 1.8)
Interpolation Buffer Settings (Section 1.63)

Parameters: None.

1.2 Integrator Settings

Description: These parameters are used directly in setting up the Integrator

Parent(s): Integrator Base (Section 1.1)

Child(ren): Integrator Selection (Section 1.3)

Parameters: **Final Time = 1**
Land On Final Time = 1

1.3 Integrator Selection

Description:

Parent(s): Integrator Settings (Section 1.2)

Child(ren): Default Integrator (Section 1.4)

Parameters: **Integrator Type = Default Integrator** Determines the type of Rythmos::Integrator object that will be built. The parameters for each Integrator Type are specified in this sub-list
Valid std::string values:
"None"
"Default Integrator"

1.4 Default Integrator

Description:

Parent(s): Integrator Selection (Section 1.3)

Child(ren): VerboseObject (Section 1.5)

Parameters: **Max Number Time Steps = 2147483647** Set the maximum number of integration time-steps allowed.

1.5 VerboseObject

Description:

Parent(s): Default Integrator (Section 1.4)
Simple Integration Control Strategy (Section 1.7)
Forward Euler (Section 1.10)
Backward Euler (Section 1.11)
Implicit BDF (Section 1.12)
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Explicit 3 Stage 3rd order by Heun (Section 1.31)
Explicit 3 Stage 3rd order TVD (Section 1.32)
Explicit 4 Stage 3rd order by Runge (Section 1.33)
Explicit 4 Stage (Section 1.34)
Explicit 3/8 Rule (Section 1.35)
Backward Euler (Section 1.36)
Singly Diagonal IRK 2 Stage 2nd order (Section 1.37)
Singly Diagonal IRK 2 Stage 3rd order (Section 1.38)
Singly Diagonal IRK 3 Stage 4th order (Section 1.39)
Singly Diagonal IRK 5 Stage 4th order (Section 1.40)
Singly Diagonal IRK 5 Stage 5th order (Section 1.41)
Diagonal IRK 2 Stage 3rd order (Section 1.42)
Implicit 1 Stage 2nd order Gauss (Section 1.43)
Implicit 2 Stage 4th order Gauss (Section 1.44)
Implicit 3 Stage 6th order Gauss (Section 1.45)
Implicit 2 Stage 4th Order Hammer & Hollingsworth (Section 1.46)
Implicit 3 Stage 6th Order Kuntzmann & Butcher (Section 1.47)
Implicit 1 Stage 1st order Radau left (Section 1.48)
Implicit 2 Stage 3rd order Radau left (Section 1.49)
Implicit 3 Stage 5th order Radau left (Section 1.50)
Implicit 1 Stage 1st order Radau right (Section 1.51)
Implicit 2 Stage 3rd order Radau right (Section 1.52)
Implicit 3 Stage 5th order Radau right (Section 1.53)
Implicit 2 Stage 2nd order Lobatto A (Section 1.54)

Implicit 3 Stage 4th order Lobatto A (Section 1.55)
 Implicit 4 Stage 6th order Lobatto A (Section 1.56)
 Implicit 2 Stage 2nd order Lobatto B (Section 1.57)
 Implicit 3 Stage 4th order Lobatto B (Section 1.58)
 Implicit 4 Stage 6th order Lobatto B (Section 1.59)
 Implicit 2 Stage 2nd order Lobatto C (Section 1.60)
 Implicit 3 Stage 4th order Lobatto C (Section 1.61)
 Implicit 4 Stage 6th order Lobatto C (Section 1.62)
 Interpolation Buffer (Section 1.65)
 Pointwise Interpolation Buffer Appender (Section 1.67)
 Linear Interpolator (Section 1.69)
 Hermite Interpolator (Section 1.70)
 Cubic Spline Interpolator (Section 1.71)

Child(ren): None.

Parameters: **Verbosity Level = default** The verbosity level to use to override whatever is set in code. The value of "default" will allow the level set in code to be used.

Valid std::string values:

"default"	Use level set in code
"none"	Produce no output
"low"	Produce minimal output
"medium"	Produce a little more output
"high"	Produce a higher level of output
"extreme"	Produce the highest level of output

Output File = none The file to send output to. If the value "none" is used, then whatever is set in code will be used. However, any other std::string value will be used to create an std::ofstream object to a file with the given name. Therefore, any valid file name is a valid std::string value for this parameter.

1.6 Integration Control Strategy Selection

Description: Note that some settings conflict between step control and integration control. In general, the integration control decides which steps will be fixed or variable, not the stepper. When the integration control decides to take variable steps, the step control is then responsible for choosing appropriate step-sizes.

Parent(s): Integrator Base (Section 1.1)

Child(ren): Simple Integration Control Strategy (Section 1.7)

Parameters: **Integration Control Strategy Type = None** Determines the type of Rythmos::IntegrationControlStrategy object that will be built. The parameters for each Integration Control Strategy Type are specified in this sublist
Valid std::string values:
"None"
"Simple Integration Control Strategy"

1.7 Simple Integration Control Strategy

Description:

Parent(s): Integration Control Strategy Selection (Section 1.6)

Child(ren): None.

Parameters: **Take Variable Steps = 1** Take variable time steps or fixed time steps. If set to false, then the parameter "Fixed dt" or "Number of Time Steps" must be set!

Max dt = 1.79769e+308 Gives the max size of the variable time steps. This is only read and used if "Take Variable Steps" is set to true.

Number of Time Steps = -1 Gives the number of fixed time steps. The actual step size gets computed on the fly given the size of the time domain. This is only read and used if "Take Variable Steps" is set to false and "Fixed dt" is set to < 0.0 .

Fixed dt = -1 Gives the size of the fixed time steps. This is only read and used if "Take Variable Steps" is set to false.

1.8 Stepper Settings

Description:

Parent(s): Integrator Base (Section 1.1)

Child(ren): Stepper Selection (Section 1.9)
Step Control Settings (Section 1.15)
Interpolator Selection (Section 1.22)
Runge Kutta Butcher Tableau Selection (Section 1.26)

Parameters: None.

1.9 Stepper Selection

Description:

Parent(s): Stepper Settings (Section 1.8)

Child(ren): Forward Euler (Section 1.10)
Backward Euler (Section 1.11)
Implicit BDF (Section 1.12)
Explicit RK (Section 1.13)
Implicit RK (Section 1.14)

Parameters: **Stepper Type = Backward Euler** Determines the type of Rythmos::Stepper object that will be built. The parameters for each Stepper Type are specified in this sublist

Valid std::string values:

"None"
"Forward Euler"
"Backward Euler"
"Implicit BDF"
"Explicit RK"
"Implicit RK"

1.10 Forward Euler

Description:

Parent(s): Stepper Selection (Section 1.9)

Child(ren): None.

Parameters: None.

1.11 Backward Euler

Description:

Parent(s): Stepper Selection (Section 1.9)

Child(ren): None.

Parameters: None.

1.12 Implicit BDF

Description:

Parent(s): Stepper Selection (Section 1.9)

Child(ren): None.

Parameters: None.

1.13 Explicit RK

Description:

Parent(s): Stepper Selection (Section 1.9)

Child(ren): None.

Parameters: None.

1.14 Implicit RK

Description:

Parent(s): Stepper Selection (Section 1.9)

Child(ren): None.

Parameters: None.

1.15 Step Control Settings

Description: Not all step control strategies are compatible with each stepper. If the strategy has the name of a stepper in its name, then it only works with that stepper.

Parent(s): Stepper Settings (Section 1.8)

Child(ren): Step Control Strategy Selection (Section 1.16)
Error Weight Vector Calculator Selection (Section 1.20)

Parameters: None.

1.16 Step Control Strategy Selection

Description:

Parent(s): Step Control Settings (Section 1.15)

Child(ren): Implicit BDF Stepper Step Control Strategy (Section 1.17)
Implicit BDF Stepper Ramping Step Control Strategy (Section 1.19)

Parameters: **Step Control Strategy Type = None** Determines the type of Rythmos::StepControlStrategy object that will be built. The parameters for each Step Control Strategy Type are specified in this sublist
Valid std::string values:
"None"
"Implicit BDF Stepper Step Control Strategy"
"Implicit BDF Stepper Ramping Step Control Strategy"

1.17 Implicit BDF Stepper Step Control Strategy

Description:

Parent(s): Step Control Strategy Selection (Section 1.16)

Child(ren): magicNumbers (Section 1.18)

Parameters: **minOrder** = **1** lower limit of order selection, guaranteed
maxOrder = **5** upper limit of order selection, does not guarantee this order
relErrTol = **0.0001**
absErrTol = **1e-06**
constantStepSize = **0**
stopTime = **10**
failStepIfNonlinearSolveFails = **0** Power user command. Will force the function `acceptStep()` to return false ieven if the LET is acceptable. Used to run with loose tolerances but enforce a correct nonlinear solution to the step.

1.18 magicNumbers

Description: These are knobs in the algorithm that have been set to reasonable values using lots of testing and heuristics and some theory.

Parent(s): Implicit BDF Stepper Step Control Strategy (Section 1.17)

Child(ren): None.

Parameters: **h0_safety** = **2**
h0_max_factor = **0.001**
h_phase0_incr = **2** initial ramp-up in variable mode (step-Size multiplier)
h_max_inv = **0**
Tkm1_Tk_safety = **2**
Tkp1_Tk_safety = **0.5**
r_factor = **0.9** used in `rejectStep`: time step ratio multiplier
r_safety = **2** local error multiplier as part of time step ratio calculation
r_fudge = **0.0001** local error addition as part of time step ratio calculation
r_min = **0.125** used in `rejectStep`: how much to cut step and lower bound for time step ratio

r_max = 0.9 upper bound for time step ratio
r_hincr_test = 2 used in completeStep: if time step ratio > this then set time step ratio to r_hincr
r_hincr = 2 used in completeStep: limit on time step ratio increases, not checked by r_max
max_LET_fail = 15 Max number of rejected steps
minTimeStep = 0 bound on smallest time step in variable mode.
maxTimeStep = 10 bound on largest time step in variable mode.

1.19 Implicit BDF Stepper Ramping Step Control Strategy

Description:

Parent(s): Step Control Strategy Selection (Section 1.16)

Child(ren): None.

Parameters: **Number of Constant First Order Steps = 10** Number of constant steps to take before handing control to variable stepper.

Initial Step Size = 0.001 Initial time step size and target step size to take during the initial constant step phase (could be reduced due to step failures).

Min Step Size = 1e-07 Minimum time step size.

Max Step Size = 1 Maximum time step size.

Step Size Increase Factor = 1.2 Time step growth factor used after a successful time step. $dt_{n+1} = (\text{increase factor}) * dt_n$

Step Size Decrease Factor = 0.5 Time step reduction factor used for a failed time step. $dt_{n+1} = (\text{decrease factor}) * dt_n$

Min Order = 1 Minimum order to run at.

Max Order = 5 Maximum order to run at.

Absolute Error Tolerance = 1e-05 abstol value used in WRMS calculation.

Relative Error Tolerance = 0.001 reltol value used in WRMS calculation.

Use **LET To Determine Step Acceptance = FALSE** If set to TRUE, then acceptance of step depends on LET in addition to Nonlinear solver converging.

Valid std::string values:

"TRUE"

"FALSE"

1.20 Error Weight Vector Calculator Selection

Description: Not all ErrWtVec calculators are compatible with each step control strategy. If the calculator has the name of a stepper or another step control strategy in its name, then it only works with that step control strategy.

Parent(s): Step Control Settings (Section 1.15)

Child(ren): Implicit BDF Stepper Error Weight Vector Calculator (Section 1.21)

Parameters: **Error Weight Vector Calculator Type = None** Determines the type of Rythmos::ErrWtVecCalc object that will be built. The parameters for each Error Weight Vector Calculator Type are specified in this sublist

Valid std::string values:

"None"

"Implicit BDF Stepper Error Weight Vector Calculator"

1.21 Implicit BDF Stepper Error Weight Vector Calculator

Description:

Parent(s): Error Weight Vector Calculator Selection (Section 1.20)

Child(ren): None.

Parameters: None.

1.22 Interpolator Selection

Description: Note all Steppers accept an interpolator. Currently, only the BackwardEuler stepper does.

Parent(s): Stepper Settings (Section 1.8)

Child(ren): Linear Interpolator (Section 1.23)
Hermite Interpolator (Section 1.24)
Cubic Spline Interpolator (Section 1.25)

Parameters: **Interpolator Type = None** Determines the type of Rythmos::Interpolator object that will be built. The parameters for each Interpolator Type are specified in this sublist

Valid std::string values:

- "None"
- "Linear Interpolator"
- "Hermite Interpolator"
- "Cubic Spline Interpolator"

1.23 Linear Interpolator

Description:

Parent(s): Interpolator Selection (Section 1.22)

Child(ren): None.

Parameters: None.

1.24 Hermite Interpolator

Description:

Parent(s): Interpolator Selection (Section 1.22)

Child(ren): None.

Parameters: None.

1.25 Cubic Spline Interpolator

Description:

Parent(s): Interpolator Selection (Section 1.22)

Child(ren): None.

Parameters: None.

1.26 Runge Kutta Butcher Tableau Selection

Description: Only the Explicit RK Stepper and the Implicit RK Stepper accept an RK Butcher Tableau.

Parent(s): Stepper Settings (Section 1.8)

Child(ren): Forward Euler (Section 1.27)
 Explicit 2 Stage 2nd order by Runge (Section 1.28)
 Explicit Trapezoidal (Section 1.29)
 Explicit 3 Stage 3rd order (Section 1.30)
 Explicit 3 Stage 3rd order by Heun (Section 1.31)
 Explicit 3 Stage 3rd order TVD (Section 1.32)
 Explicit 4 Stage 3rd order by Runge (Section 1.33)
 Explicit 4 Stage (Section 1.34)
 Explicit 3/8 Rule (Section 1.35)
 Backward Euler (Section 1.36)
 Singly Diagonal IRK 2 Stage 2nd order (Section 1.37)
 Singly Diagonal IRK 2 Stage 3rd order (Section 1.38)
 Singly Diagonal IRK 3 Stage 4th order (Section 1.39)
 Singly Diagonal IRK 5 Stage 4th order (Section 1.40)
 Singly Diagonal IRK 5 Stage 5th order (Section 1.41)
 Diagonal IRK 2 Stage 3rd order (Section 1.42)
 Implicit 1 Stage 2nd order Gauss (Section 1.43)
 Implicit 2 Stage 4th order Gauss (Section 1.44)
 Implicit 3 Stage 6th order Gauss (Section 1.45)
 Implicit 2 Stage 4th Order Hammer & Hollingsworth (Section 1.46)
 Implicit 3 Stage 6th Order Kuntzmann & Butcher (Section 1.47)
 Implicit 1 Stage 1st order Radau left (Section 1.48)
 Implicit 2 Stage 3rd order Radau left (Section 1.49)
 Implicit 3 Stage 5th order Radau left (Section 1.50)
 Implicit 1 Stage 1st order Radau right (Section 1.51)
 Implicit 2 Stage 3rd order Radau right (Section 1.52)
 Implicit 3 Stage 5th order Radau right (Section 1.53)
 Implicit 2 Stage 2nd order Lobatto A (Section 1.54)
 Implicit 3 Stage 4th order Lobatto A (Section 1.55)
 Implicit 4 Stage 6th order Lobatto A (Section 1.56)
 Implicit 2 Stage 2nd order Lobatto B (Section 1.57)
 Implicit 3 Stage 4th order Lobatto B (Section 1.58)
 Implicit 4 Stage 6th order Lobatto B (Section 1.59)
 Implicit 2 Stage 2nd order Lobatto C (Section 1.60)
 Implicit 3 Stage 4th order Lobatto C (Section 1.61)
 Implicit 4 Stage 6th order Lobatto C (Section 1.62)

Parameters: **Runge Kutta Butcher Tableau Type = None** Determines the type of Rythmos::RKButcherTableau object that will be built. The parameters for each Runge Kutta Butcher Tableau Type are specified in this sublist
 Valid std::string values:

"None"
 "Forward Euler"
 "Explicit 2 Stage 2nd order by Runge"
 "Explicit Trapezoidal"
 "Explicit 3 Stage 3rd order"
 "Explicit 3 Stage 3rd order by Heun"
 "Explicit 3 Stage 3rd order TVD"
 "Explicit 4 Stage 3rd order by Runge"
 "Explicit 4 Stage"
 "Explicit 3/8 Rule"
 "Backward Euler"
 "Singly Diagonal IRK 2 Stage 2nd order"
 "Singly Diagonal IRK 2 Stage 3rd order"
 "Singly Diagonal IRK 3 Stage 4th order"
 "Singly Diagonal IRK 5 Stage 4th order"
 "Singly Diagonal IRK 5 Stage 5th order"
 "Diagonal IRK 2 Stage 3rd order"
 "Implicit 1 Stage 2nd order Gauss"
 "Implicit 2 Stage 4th order Gauss"
 "Implicit 3 Stage 6th order Gauss"
 "Implicit 2 Stage 4th Order Hammer & Hollingsworth"
 "Implicit 3 Stage 6th Order Kuntzmann & Butcher"
 "Implicit 1 Stage 1st order Radau left"
 "Implicit 2 Stage 3rd order Radau left"
 "Implicit 3 Stage 5th order Radau left"
 "Implicit 1 Stage 1st order Radau right"
 "Implicit 2 Stage 3rd order Radau right"
 "Implicit 3 Stage 5th order Radau right"
 "Implicit 2 Stage 2nd order Lobatto A"
 "Implicit 3 Stage 4th order Lobatto A"
 "Implicit 4 Stage 6th order Lobatto A"
 "Implicit 2 Stage 2nd order Lobatto B"
 "Implicit 3 Stage 4th order Lobatto B"
 "Implicit 4 Stage 6th order Lobatto B"
 "Implicit 2 Stage 2nd order Lobatto C"
 "Implicit 3 Stage 4th order Lobatto C"
 "Implicit 4 Stage 6th order Lobatto C"

1.27 Forward Euler

Description: **Forward Euler**
 $c = [0]'$
 $A = [0]$
 $b = [1]'$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.28 Explicit 2 Stage 2nd order by Runge

Description: Explicit 2 Stage 2nd order by Runge
Also known as Explicit Midpoint
Solving Ordinary Differential Equations I:
Nonstiff Problems, 2nd Revised Edition
E. Hairer, S.P. Norsett, G. Wanner
Table 1.1, pg 135
 $c = \begin{bmatrix} 0 & 1/2 \end{bmatrix}'$
 $A = \begin{bmatrix} 0 & \\ 1/2 & 0 \end{bmatrix}$
 $b = \begin{bmatrix} 0 & 1 \end{bmatrix}'$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.29 Explicit Trapezoidal

Description: Explicit Trapezoidal
 $c = \begin{bmatrix} 0 & 1 \end{bmatrix}'$
 $A = \begin{bmatrix} 0 & \\ 1 & 0 \end{bmatrix}$
 $b = \begin{bmatrix} 1/2 & 1/2 \end{bmatrix}'$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.30 Explicit 3 Stage 3rd order

Description: Explicit 3 Stage 3rd order
 $c = \begin{bmatrix} 0 & 1/2 & 1 \end{bmatrix}'$
 $A = \begin{bmatrix} 0 & & \\ 1/2 & 0 & \\ -1 & 2 & 0 \end{bmatrix}$
 $b = \begin{bmatrix} 1/6 & 4/6 & 1/6 \end{bmatrix}'$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.31 Explicit 3 Stage 3rd order by Heun

Description: Explicit 3 Stage 3rd order by Heun
Solving Ordinary Differential Equations I:
Nonstiff Problems, 2nd Revised Edition
E. Hairer, S.P. Norsett, G. Wanner
Table 1.1, pg 135
 $c = [0 \quad 1/3 \quad 2/3]'$
 $A = [\begin{array}{ccc} 0 & & \\ 1/3 & 0 & \\ 0 & 2/3 & 0 \end{array}]$
 $b = [1/4 \quad 0 \quad 3/4]'$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.32 Explicit 3 Stage 3rd order TVD

Description: Explicit 3 Stage 3rd order TVD
Sigal Gottlieb and Chi-Wang Shu
'Total Variation Diminishing Runge-Kutta Schemes'
Mathematics of Computation
Volume 67, Number 221, January 1998, pp. 73-85
 $c = [0 \quad 1 \quad 1/2]'$
 $A = [\begin{array}{ccc} 0 & & \\ 1 & 0 & \\ 1/4 & 1/4 & 0 \end{array}]$
 $b = [1/6 \quad 1/6 \quad 4/6]'$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.33 Explicit 4 Stage 3rd order by Runge

Description: Explicit 4 Stage 3rd order by Runge
Solving Ordinary Differential Equations I:
Nonstiff Problems, 2nd Revised Edition
E. Hairer, S.P. Norsett, G. Wanner
Table 1.1, pg 135
 $c = [0 \quad 1/2 \quad 1 \quad 1]'$
 $A = [\begin{array}{cccc} 0 & & & \\ 1/2 & 0 & & \\ 0 & 1 & 0 & \end{array}]$

$$\mathbf{b} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1/6 & 2/3 & 0 & 1/6 \end{bmatrix},$$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.34 Explicit 4 Stage

Description: Explicit 4 Stage
 "The" Runge-Kutta Method (explicit):
 Solving Ordinary Differential Equations I:
 Nonstiff Problems, 2nd Revised Edition
 E. Hairer, S.P. Norsett, G. Wanner
 Table 1.2, pg 138
 $\mathbf{c} = \begin{bmatrix} 0 & 1/2 & 1/2 & 1 \end{bmatrix},$
 $\mathbf{A} = \begin{bmatrix} 0 & & & \\ 1/2 & 0 & & \\ 0 & 1/2 & 0 & \\ 0 & 0 & 1 & 0 \end{bmatrix}$
 $\mathbf{b} = \begin{bmatrix} 1/6 & 1/3 & 1/3 & 1/6 \end{bmatrix},$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.35 Explicit 3/8 Rule

Description: Explicit 3/8 Rule
 Solving Ordinary Differential Equations I:
 Nonstiff Problems, 2nd Revised Edition
 E. Hairer, S.P. Norsett, G. Wanner
 Table 1.2, pg 138
 $\mathbf{c} = \begin{bmatrix} 0 & 1/3 & 2/3 & 1 \end{bmatrix},$
 $\mathbf{A} = \begin{bmatrix} 0 & & & \\ 1/3 & 0 & & \\ -1/3 & 1 & 0 & \\ 1 & -1 & 1 & 0 \end{bmatrix}$
 $\mathbf{b} = \begin{bmatrix} 1/8 & 3/8 & 3/8 & 1/8 \end{bmatrix},$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.36 Backward Euler

Description: Backward Euler
 $c = [1]$
 $A = [1]$
 $b = [1]$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.37 Singly Diagonal IRK 2 Stage 2nd order

Description: Singly Diagonal IRK 2 Stage 2nd order
Computer Methods for ODEs and DAEs
U. M. Ascher and L. R. Petzold
p. 106
 $\gamma = (2 + \sqrt{2})/2$
 $c = [\gamma, 1]$
 $A = [\gamma, 0]$
 $[1 - \gamma, \gamma]$
 $b = [1 - \gamma, \gamma]$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: **gamma = 0.292893** The default value is $\gamma = (2 + \sqrt{2})/2 = 0.29289321881345243$. This will produce an L-stable 2nd order method with the stage times within the timestep. Other values of γ will still produce an L-stable scheme, but will only be 1st order accurate.

1.38 Singly Diagonal IRK 2 Stage 3rd order

Description: Singly Diagonal IRK 2 Stage 3rd order
Solving Ordinary Differential Equations I:
Nonstiff Problems, 2nd Revised Edition
E. Hairer, S. P. Norsett, and G. Wanner
Table 7.2, pg 207
 $\gamma = (3 + \sqrt{3})/6$ -> 3rd order and A-stable
 $\gamma = (2 + \sqrt{2})/2$ -> 2nd order and L-stable
 $c = [\gamma, 1 - \gamma]$
 $A = [\gamma, 0]$
 $[1 - 2\gamma, \gamma]$
 $b = [1/2, 1/2]$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: **3rd Order A-stable = 1** If true, set gamma to $\gamma = (3+\sqrt{3})/6$ to obtain a 3rd order A-stable scheme. '3rd Order A-stable' and '2nd Order L-stable' can not both be true.

2nd Order L-stable = 0 If true, set gamma to $\gamma = (2+\sqrt{2})/2$ to obtain a 2nd order L-stable scheme. '3rd Order A-stable' and '2nd Order L-stable' can not both be true.

gamma = 0.788675 If both '3rd Order A-stable' and '2nd Order L-stable' are false, gamma will be used. The default value is the '3rd Order A-stable' gamma value, $(3+\sqrt{3})/6$.

1.39 Singly Diagonal IRK 3 Stage 4th order

Description: Singly Diagonal IRK 3 Stage 4th order
A-stable
Solving Ordinary Differential Equations II:
Stiff and Differential-Algebraic Problems,
2nd Revised Edition
E. Hairer and G. Wanner
pg100
 $\gamma = (1/\sqrt{3})*\cos(\pi/18)+1/2$
 $\delta = 1/(6*(2*\gamma-1)^2)$
 $c = [\gamma \quad 1/2 \quad 1-\gamma]$,
 $A = [\gamma \quad \quad \quad]$
 $\quad [1/2-\gamma \quad \gamma \quad \quad]$
 $\quad [2*\gamma \quad 1-4*\gamma \quad \gamma]$
 $b = [\delta \quad 1-2*\delta \quad \delta]$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.40 Singly Diagonal IRK 5 Stage 4th order

Description: Singly Diagonal IRK 5 Stage 4th order
L-stable
Solving Ordinary Differential Equations II:
Stiff and Differential-Algebraic Problems,
2nd Revised Edition
E. Hairer and G. Wanner

pg100
 $c = [\begin{array}{ccccc} 1/4 & 3/4 & 11/20 & 1/2 & 1 \end{array}]'$
 $A = [\begin{array}{ccccc} 1/4 & & & & \\ & 1/2 & 1/4 & & \\ & 17/50 & -1/25 & 1/4 & \\ & 371/1360 & -137/2720 & 15/544 & 1/4 \\ & 25/24 & -49/48 & 125/16 & -85/12 \end{array}]$
 $b = [\begin{array}{ccccc} 25/24 & -49/48 & 125/16 & -85/12 & 1/4 \end{array}]'$
 $b' = [\begin{array}{ccccc} 59/48 & -17/96 & 225/32 & -85/12 & 0 \end{array}]'$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.41 Singly Diagonal IRK 5 Stage 5th order

Description: Singly Diagonal IRK 5 Stage 5th order
A-stable
Solving Ordinary Differential Equations II:
Stiff and Differential-Algebraic Problems,
2nd Revised Edition
E. Hairer and G. Wanner

pg101
 $c = [\begin{array}{ccc} (6-\sqrt{6})/10 & (6+9\sqrt{6})/35 & 1 \end{array}]$
 $A = [\begin{array}{ccc} (6-\sqrt{6})/10 & & \\ & (-6+5\sqrt{6})/14 & (6-\sqrt{6})/10 \\ & (888+607\sqrt{6})/2850 & (126-161\sqrt{6})/1425 & (6-\sqrt{6})/10 \\ & (3153-3082\sqrt{6})/14250 & (3213+1148\sqrt{6})/28500 & (-267+88\sqrt{6})/14250 \\ & (-32583+14638\sqrt{6})/71250 & (-17199+364\sqrt{6})/142500 & (1329-54\sqrt{6})/71250 \end{array}]$
 $b = [\begin{array}{ccc} 0 & 0 & 1/9 \end{array}]$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.42 Diagonal IRK 2 Stage 3rd order

Description: Diagonal IRK 2 Stage 3rd order
Hammer & Hollingsworth method
Solving Ordinary Differential Equations I:
Nonstiff Problems, 2nd Revised Edition
E. Hairer, S. P. Norsett, and G. Wanner
Table 7.1, pg 205
 $c = [\begin{array}{cc} 0 & 2/3 \end{array}]'$

$$A = \begin{bmatrix} 0 & 0 \\ 1/3 & 1/3 \end{bmatrix}$$

$$b = \begin{bmatrix} 1/4 & 3/4 \end{bmatrix},$$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.43 Implicit 1 Stage 2nd order Gauss

Description: Implicit 1 Stage 2nd order Gauss
A-stable
Solving Ordinary Differential Equations II:
Stiff and Differential-Algebraic Problems,
2nd Revised Edition
E. Hairer and G. Wanner
Table 5.2, pg 72
Also: Implicit midpoint rule
Solving Ordinary Differential Equations I:
Nonstiff Problems, 2nd Revised Edition
E. Hairer, S. P. Norsett, and G. Wanner
Table 7.1, pg 205
 $c = \begin{bmatrix} 1/2 \end{bmatrix}$,
 $A = \begin{bmatrix} 1/2 \end{bmatrix}$
 $b = \begin{bmatrix} 1 \end{bmatrix}$,

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.44 Implicit 2 Stage 4th order Gauss

Description: Implicit 2 Stage 4th order Gauss
A-stable
Solving Ordinary Differential Equations II:
Stiff and Differential-Algebraic Problems,
2nd Revised Edition
E. Hairer and G. Wanner
Table 5.2, pg 72
 $c = \begin{bmatrix} 1/2-\sqrt{3}/6 & 1/2+\sqrt{3}/6 \end{bmatrix}$,
 $A = \begin{bmatrix} 1/4 & 1/4-\sqrt{3}/6 \\ 1/4+\sqrt{3}/6 & 1/4 \end{bmatrix}$
 $b = \begin{bmatrix} 1/2 & 1/2 \end{bmatrix}$,

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.45 Implicit 3 Stage 6th order Gauss

Description: Implicit 3 Stage 6th order Gauss
A-stable
Solving Ordinary Differential Equations II:
Stiff and Differential-Algebraic Problems,
2nd Revised Edition
E. Hairer and G. Wanner
Table 5.2, pg 72
$$c = \begin{bmatrix} 1/2 - \sqrt{15}/10 & 1/2 & 1/2 + \sqrt{15}/10 \end{bmatrix},$$
$$A = \begin{bmatrix} 5/36 & 2/9 - \sqrt{15}/15 & 5/36 - \sqrt{15}/30 \\ 5/36 + \sqrt{15}/24 & 2/9 & 5/36 - \sqrt{15}/24 \\ 5/36 + \sqrt{15}/30 & 2/9 + \sqrt{15}/15 & 5/36 \end{bmatrix}$$
$$b = \begin{bmatrix} 5/18 & 4/9 & 5/18 \end{bmatrix},$$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.46 Implicit 2 Stage 4th Order Hammer & Hollingsworth

Description: Implicit 2 Stage 4th Order Hammer & Hollingsworth
Hammer & Hollingsworth method
Solving Ordinary Differential Equations I:
Nonstiff Problems, 2nd Revised Edition
E. Hairer, S. P. Norsett, and G. Wanner
Table 7.3, pg 207
$$c = \begin{bmatrix} 1/2 - \sqrt{3}/6 & 1/2 + \sqrt{3}/6 \end{bmatrix},$$
$$A = \begin{bmatrix} 1/4 & 1/4 - \sqrt{3}/6 \\ 1/4 + \sqrt{3}/6 & 1/4 \end{bmatrix}$$
$$b = \begin{bmatrix} 1/2 & 1/2 \end{bmatrix},$$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.47 Implicit 3 Stage 6th Order Kuntzmann & Butcher

Description: Implicit 3 Stage 6th Order Kuntzmann & Butcher
Kuntzmann & Butcher method
Solving Ordinary Differential Equations I:

Nonstiff Problems, 2nd Revised Edition
 E. Hairer, S. P. Norsett, and G. Wanner
 Table 7.4, pg 209

$$c = \begin{bmatrix} 1/2 - \sqrt{15}/10 & 1/2 & 1/2 - \sqrt{15}/10 \end{bmatrix},$$

$$A = \begin{bmatrix} 5/36 & 2/9 - \sqrt{15}/15 & 5/36 - \sqrt{15}/30 \\ 5/36 + \sqrt{15}/24 & 2/9 & 5/36 - \sqrt{15}/24 \\ 5/36 + \sqrt{15}/30 & 2/9 + \sqrt{15}/15 & 5/36 \end{bmatrix}$$

$$b = \begin{bmatrix} 5/18 & 4/9 & 5/18 \end{bmatrix},$$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.48 Implicit 1 Stage 1st order Radau left

Description: Implicit 1 Stage 1st order Radau left
 A-stable
 Solving Ordinary Differential Equations II:
 Stiff and Differential-Algebraic Problems,
 2nd Revised Edition
 E. Hairer and G. Wanner
 Table 5.3, pg 73

$$c = \begin{bmatrix} 0 \end{bmatrix},$$

$$A = \begin{bmatrix} 1 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \end{bmatrix},$$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.49 Implicit 2 Stage 3rd order Radau left

Description: Implicit 2 Stage 3rd order Radau left
 A-stable
 Solving Ordinary Differential Equations II:
 Stiff and Differential-Algebraic Problems,
 2nd Revised Edition
 E. Hairer and G. Wanner
 Table 5.3, pg 73

$$c = \begin{bmatrix} 0 & 2/3 \end{bmatrix},$$

$$A = \begin{bmatrix} 1/4 & -1/4 \\ 1/4 & 5/12 \end{bmatrix}$$

$$b = \begin{bmatrix} 1/4 & 3/4 \end{bmatrix},$$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.50 Implicit 3 Stage 5th order Radau left

Description: Implicit 3 Stage 5th order Radau left
A-stable
Solving Ordinary Differential Equations II:
Stiff and Differential-Algebraic Problems,
2nd Revised Edition
E. Hairer and G. Wanner
Table 5.4, pg 73
$$c = \begin{bmatrix} 0 & (6-\sqrt{6})/10 & (6+\sqrt{6})/10 \end{bmatrix},$$
$$A = \begin{bmatrix} 1/9 & (-1-\sqrt{6})/18 & (-1+\sqrt{6})/18 \\ 1/9 & (88+7\sqrt{6})/360 & (88-43\sqrt{6})/360 \\ 1/9 & (88+43\sqrt{6})/360 & (88-7\sqrt{6})/360 \end{bmatrix}$$
$$b = \begin{bmatrix} 1/9 & (16+\sqrt{6})/36 & (16-\sqrt{6})/36 \end{bmatrix},$$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.51 Implicit 1 Stage 1st order Radau right

Description: Implicit 1 Stage 1st order Radau right
A-stable
Solving Ordinary Differential Equations II:
Stiff and Differential-Algebraic Problems,
2nd Revised Edition
E. Hairer and G. Wanner
Table 5.5, pg 74
$$c = \begin{bmatrix} 1 \end{bmatrix},$$
$$A = \begin{bmatrix} 1 \end{bmatrix}$$
$$b = \begin{bmatrix} 1 \end{bmatrix},$$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.52 Implicit 2 Stage 3rd order Radau right

Description: Implicit 2 Stage 3rd order Radau right
A-stable
Solving Ordinary Differential Equations II:

Stiff and Differential-Algebraic Problems,
 2nd Revised Edition
 E. Hairer and G. Wanner
 Table 5.5, pg 74
 $c = \begin{bmatrix} 1/3 & 1 \end{bmatrix}'$
 $A = \begin{bmatrix} 5/12 & -1/12 \\ 3/4 & 1/4 \end{bmatrix}$
 $b = \begin{bmatrix} 3/4 & 1/4 \end{bmatrix}'$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.53 Implicit 3 Stage 5th order Radau right

Description: Implicit 3 Stage 5th order Radau right
 A-stable
 Solving Ordinary Differential Equations II:
 Stiff and Differential-Algebraic Problems,
 2nd Revised Edition
 E. Hairer and G. Wanner
 Table 5.6, pg 74
 $c = \begin{bmatrix} (4-\sqrt{6})/10 & (4+\sqrt{6})/10 & 1 \end{bmatrix}$
 $A = \begin{bmatrix} (88-7\sqrt{6})/360 & (296-169\sqrt{6})/1800 & (-2+3\sqrt{6})/225 \\ (296+169\sqrt{6})/1800 & (88+7\sqrt{6})/360 & (-2-3\sqrt{6})/225 \\ (16-\sqrt{6})/36 & (16+\sqrt{6})/36 & 1/9 \end{bmatrix}$
 $b = \begin{bmatrix} (16-\sqrt{6})/36 & (16+\sqrt{6})/36 & 1/9 \end{bmatrix}$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.54 Implicit 2 Stage 2nd order Lobatto A

Description: Implicit 2 Stage 2nd order Lobatto A
 A-stable
 Solving Ordinary Differential Equations II:
 Stiff and Differential-Algebraic Problems,
 2nd Revised Edition
 E. Hairer and G. Wanner
 Table 5.7, pg 75
 $c = \begin{bmatrix} 0 & 1 \end{bmatrix}'$
 $A = \begin{bmatrix} 0 & 0 \\ 1/2 & 1/2 \end{bmatrix}$
 $b = \begin{bmatrix} 1/2 & 1/2 \end{bmatrix}'$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.55 Implicit 3 Stage 4th order Lobatto A

Description: Implicit 3 Stage 4th order Lobatto A
A-stable
Solving Ordinary Differential Equations II:
Stiff and Differential-Algebraic Problems,
2nd Revised Edition
E. Hairer and G. Wanner
Table 5.7, pg 75
 $c = [0 \quad 1/2 \quad 1]'$
 $A = [0 \quad 0 \quad 0]$
 $\quad [5/24 \quad 1/3 \quad -1/24]$
 $\quad [1/6 \quad 2/3 \quad 1/6]$
 $b = [1/6 \quad 2/3 \quad 1/6]'$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.56 Implicit 4 Stage 6th order Lobatto A

Description: Implicit 4 Stage 6th order Lobatto A
A-stable
Solving Ordinary Differential Equations II:
Stiff and Differential-Algebraic Problems,
2nd Revised Edition
E. Hairer and G. Wanner
Table 5.8, pg 75
 $c = [0 \quad (5-\sqrt{5})/10 \quad (5+\sqrt{5})/10 \quad 1]$
 $A = [0 \quad 0 \quad 0 \quad 0]$
 $\quad [(11+\sqrt{5})/120 \quad (25-\sqrt{5})/120 \quad (25-13\sqrt{5})/120 \quad (-1+\sqrt{5})/120]$
 $\quad [(11-\sqrt{5})/120 \quad (25+13\sqrt{5})/120 \quad (25+\sqrt{5})/120 \quad (-1-\sqrt{5})/120]$
 $\quad [1/12 \quad 5/12 \quad 5/12 \quad 1/12]$
 $b = [1/12 \quad 5/12 \quad 5/12 \quad 1/12]'$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.57 Implicit 2 Stage 2nd order Lobatto B

Description: Implicit 2 Stage 2nd order Lobatto B
A-stable
Solving Ordinary Differential Equations II:
Stiff and Differential-Algebraic Problems,
2nd Revised Edition
E. Hairer and G. Wanner
Table 5.9, pg 76
 $c = \begin{bmatrix} 0 & 1 \end{bmatrix}$,
 $A = \begin{bmatrix} 1/2 & 0 \\ 1/2 & 0 \end{bmatrix}$
 $b = \begin{bmatrix} 1/2 & 1/2 \end{bmatrix}$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.58 Implicit 3 Stage 4th order Lobatto B

Description: Implicit 3 Stage 4th order Lobatto B
A-stable
Solving Ordinary Differential Equations II:
Stiff and Differential-Algebraic Problems,
2nd Revised Edition
E. Hairer and G. Wanner
Table 5.9, pg 76
 $c = \begin{bmatrix} 0 & 1/2 & 1 \end{bmatrix}$,
 $A = \begin{bmatrix} 1/6 & -1/6 & 0 \\ 1/6 & 1/3 & 0 \\ 1/6 & 5/6 & 0 \end{bmatrix}$
 $b = \begin{bmatrix} 1/6 & 2/3 & 1/6 \end{bmatrix}$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.59 Implicit 4 Stage 6th order Lobatto B

Description: Implicit 4 Stage 6th order Lobatto B
A-stable
Solving Ordinary Differential Equations II:
Stiff and Differential-Algebraic Problems,
2nd Revised Edition
E. Hairer and G. Wanner

Table 5.10, pg 76

$$\begin{aligned}
 c &= [0 \quad (5-\sqrt{5})/10 \quad (5+\sqrt{5})/10 \quad 1]' \\
 A &= [1/12 \quad (-1-\sqrt{5})/24 \quad (-1+\sqrt{5})/24 \quad 0] \\
 &\quad [1/12 \quad (25+\sqrt{5})/120 \quad (25-13\sqrt{5})/120 \quad 0] \\
 &\quad [1/12 \quad (25+13\sqrt{5})/120 \quad (25-\sqrt{5})/120 \quad 0] \\
 &\quad [1/12 \quad (11-\sqrt{5})/24 \quad (11+\sqrt{5})/24 \quad 0] \\
 b &= [1/12 \quad 5/12 \quad 5/12 \quad 1/12]'
 \end{aligned}$$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.60 Implicit 2 Stage 2nd order Lobatto C

Description: Implicit 2 Stage 2nd order Lobatto C
A-stable
Solving Ordinary Differential Equations II:
Stiff and Differential-Algebraic Problems,
2nd Revised Edition
E. Hairer and G. Wanner
Table 5.11, pg 76
 $c = [0 \quad 1]'$
 $A = [1/2 \quad -1/2]$
 $\quad [1/2 \quad 1/2]$
 $b = [1/2 \quad 1/2]'$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.61 Implicit 3 Stage 4th order Lobatto C

Description: Implicit 3 Stage 4th order Lobatto C
A-stable
Solving Ordinary Differential Equations II:
Stiff and Differential-Algebraic Problems,
2nd Revised Edition
E. Hairer and G. Wanner
Table 5.11, pg 76
 $c = [0 \quad 1/2 \quad 1]'$
 $A = [1/6 \quad -1/3 \quad 1/6]$
 $\quad [1/6 \quad 5/12 \quad -1/12]$
 $\quad [1/6 \quad 2/3 \quad 1/6]$
 $b = [1/6 \quad 2/3 \quad 1/6]'$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.62 Implicit 4 Stage 6th order Lobatto C

Description: Implicit 4 Stage 6th order Lobatto C
A-stable
Solving Ordinary Differential Equations II:
Stiff and Differential-Algebraic Problems,
2nd Revised Edition
E. Hairer and G. Wanner
Table 5.12, pg 76
$$c = \begin{bmatrix} 0 & (5-\sqrt{5})/10 & (5+\sqrt{5})/10 & 1 \end{bmatrix},$$
$$A = \begin{bmatrix} 1/12 & -\sqrt{5}/12 & \sqrt{5}/12 & -1/12 \\ 1/12 & 1/4 & (10-7\sqrt{5})/60 & \sqrt{5}/60 \\ 1/12 & (10+7\sqrt{5})/60 & 1/4 & -\sqrt{5}/60 \\ 1/12 & 5/12 & 5/12 & 1/12 \end{bmatrix}$$
$$b = \begin{bmatrix} 1/12 & 5/12 & 5/12 & 1/12 \end{bmatrix},$$

Parent(s): Runge Kutta Butcher Tableau Selection (Section 1.26)

Child(ren): None.

Parameters: None.

1.63 Interpolation Buffer Settings

Description:

Parent(s): Integrator Base (Section 1.1)

Child(ren): Trailing Interpolation Buffer Selection (Section 1.64)
Interpolation Buffer Appender Selection (Section 1.66)
Interpolator Selection (Section 1.68)

Parameters: None.

1.64 Trailing Interpolation Buffer Selection

Description:

Parent(s): Interpolation Buffer Settings (Section 1.63)

Child(ren): Interpolation Buffer (Section 1.65)

Parameters: **Interpolation Buffer Type = None** Determines the type of Rythmos::InterpolationBuffer object that will be built. The parameters for each Interpolation Buffer Type are specified in this sublist
Valid std::string values:
"None"
"Interpolation Buffer"

1.65 Interpolation Buffer

Description:

Parent(s): Trailing Interpolation Buffer Selection (Section 1.64)

Child(ren): None.

Parameters: **InterpolationBufferPolicy = Keep Newest Policy** Interpolation Buffer Policy for when the maximum storage size is exceeded. Static will throw an exception when the storage limit is exceeded. Keep Newest will over-write the oldest data in the buffer when the storage limit is exceeded.
Valid std::string values:
"Invalid Policy"
"Static Policy"
"Keep Newest Policy"

StorageLimit = 0 Storage limit for the interpolation buffer.

1.66 Interpolation Buffer Appender Selection

Description:

Parent(s): Interpolation Buffer Settings (Section 1.63)

Child(ren): Pointwise Interpolation Buffer Appender (Section 1.67)

Parameters: **Interpolation Buffer Appender Type = None** Determines the type of Rythmos::InterpolationBufferAppender object that will be built. The parameters for each Interpolation Buffer Appender Type are specified in this sublist
Valid std::string values:
"None"
"Pointwise Interpolation Buffer Appender"

1.67 Pointwise Interpolation Buffer Appender

Description:

Parent(s): Interpolation Buffer Appender Selection (Section 1.66)

Child(ren): None.

Parameters: None.

1.68 Interpolator Selection

Description:

Parent(s): Interpolation Buffer Settings (Section 1.63)

Child(ren): Linear Interpolator (Section 1.69)
Hermite Interpolator (Section 1.70)
Cubic Spline Interpolator (Section 1.71)

Parameters: **Interpolator Type = None** Determines the type of Rythmos::Interpolator object that will be built. The parameters for each Interpolator Type are specified in this sublist
Valid std::string values:

"None"

"Linear Interpolator"

"Hermite Interpolator"

"Cubic Spline Interpolator"

1.69 Linear Interpolator

Description:

Parent(s): Interpolator Selection (Section 1.68)

Child(ren): None.

Parameters: None.

1.70 Hermite Interpolator

Description:

Parent(s): Interpolator Selection (Section 1.68)

Child(ren): None.

Parameters: None.

1.71 Cubic Spline Interpolator

Description:

Parent(s): Interpolator Selection (Section 1.68)

Child(ren): None.

Parameters: None.